ABSOLUTE DETERMINATION OF STRESS IN TEXTURED MATERIALS

R. B. Thompson, J. F. Smith, S. S. Lee

Ames Laboratory, USDOE
Iowa State University
Ames, IA 50011

ABSTRACT

The continuum theory of elastic wave propagation in deformed, anisotropic solids is reviewed with emphasis on those features which might be used to distinguish between stress induced changes in ultrasonic velocity and changes due to material anisotropy, such as would be produced by preferred grain orientation in a polycrystalline metal. As noted by previous authors, one such feature is the difference in velocity of two shear waves, whose directions of propagation and polarization have been interchanged. In particular, when these directions fall along the symmetry axes of a rolled plate (assuming orthorhombic symmetry) and these are also the directions of principal stress, then the theory predicts that $\rho(V_{12}^2-V_{21}^2) = T_1-T_2$ where ρ is the density, V_{ij} is the velocity of a shear wave propagating along the i-axis and polarized along the j-axis, and T_i is a principal stress component. In addition to being independent of the degree of texture, this relationship has the advantage that no microstructurally dependent acoustoelastic coefficient is involved. The applicability of this prediction of continuum theory to heterogeneous engineering materials such as metal polycrystals is discussed using previously reported stress dependencies of ultrasonic velocities, and new experiments to answer some remaining questions are described. A possible configuration for using the effect to measure the value of a uniform stress in a plate of unknown texture is proposed.

INTRODUCTION

The acoustoelastic measurement of stress has the capability of sensing stresses (or the associated strains) within the interior of an elastic body. This differentiates the acoustoelastic approach from most other techniques, such as x-ray diffraction, which only sense stress induced deformations in a near surface layer. However, a number of fundamental problems have prevented the widespread practical application of the ultrasonic technique. In addition to the precision required to measure the small, stress induced velocity shifts, there are two other classes of problems. A complete description of the stress (or strain) state requires measurement of six independent tensor components as a function of spatial position. Multiple beam measurements and tomographic reconstruction techniques can be used to address this problem, but such approaches will not be discussed here. This paper is concerned with the second problem, that of differentiating stress induced velocity shifts from velocity shifts induced by microstructural variations.

Figure 1 schematically illustrates the nature of this problem. Stress is determined on the basis of an assumed linear relationship between the velocity of an ultrasonic wave and the stress in the material. If one uniquely knows this relationship, as can be determined from a calibration experiment, then the determination of the value of stress is unambiguous. (Throughout this paper, it will always be assumed that the stress is spatially uniform over the dimensions of the measurement.) Such a case is illustrated by the dotted lines intersecting the solid curve. However, in practice, one would often like to measure the stress in a structure, and it is impossible to determine the calibration curve for the material in question. Two difficulties can then ensue. First, the ultrasonic velocity at zero stress can be shifted due to a preferred orientation of elastically anisotropic grains. Such texturing, produced for example by rolling, can easily produce velocity variations of a few percent or more, which can be comparable to or greater than the velocity shifts that would be produced by the yield stress. This case is illustrated by the dashed line. Furthermore, the slope of the curve can also vary substantially[1]. Because of these two microstructurally dependent velocity shifts, the stress corresponding to a measured velocity can have a large uncertainty, as indicated in Fig. 1. As will be evident from subsequent discussion, the same comments apply to the ultrasonic birefringence test.

A possible key to the solution of this problem has been discussed in the literature. In 1940, Biott[2] noted that "The propagation of elastic waves in a material under initial stress is fundamentally different from the stress free case...follows laws which cannot be explained by elastic anisotropy or changes in elastic constants." The point was discussed in greater detail in 1965 by Thurston[3], who stated that "A transverse wave propagates faster in the direction of tension than in the perpendicular direction..., just as in a stretched string. The difference in ρV^2 for the two waves equals the tensile stress." The possibility of using this effect for the separation of the effects of stress and texture was

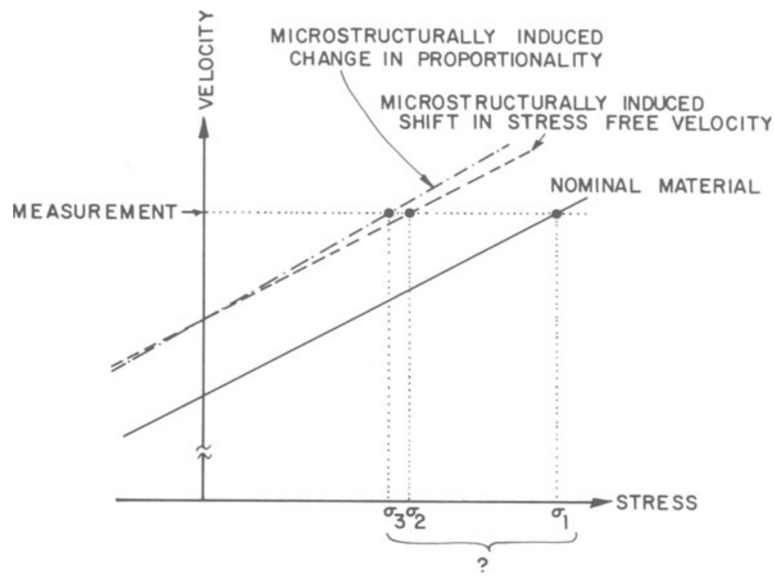

Fig. 1. Microstructurally related uncertainties in the inference of stress from ultrasonic velocity measurements.

proposed in 1981 by MacDonald[4], who suggested that "A comparison of wave speeds...indicates how to separate the effects of stress and texture."

This theoretically proposed relationship between the velocity of two shear waves propagating in perpendicular directions, and the stress in the material, appears to have the potential of being a key element in overcoming the aforementioned, microstructurally related ambiguities in the acoustoelastic technique. The technical issues involved in the application of this relationship are the correct interpretation of the predictions of the continuum theory, the determination of its applicability to engineering materials (i.e., polycrystals rather than an idealized anisotropic elastic continuum), and the identification of practical configurations whereby this effect could be used to measure stress in a structure of interest. The points are addressed in the subsequent sections of this paper.

INTERPRETATION OF CONTINUUM THEORY

When a static load is applied to a material, the transit time of an ultrasonic wave through the material changes due to a variety of reasons. The load induced deformation will alter the density and interatomic forces in the material and thereby modify the speed of the ultrasonic wave. These dimensional changes of the sample will further influence the observable transit time of an ultrasonic wave across the sample by altering the propagation path length and the differential cross-sectional areas to which the dynamic stresses of the ultrasonic wave are applied. Furthermore, as implied in the previous section, there is the possibility of a direct effect of the applied stress on the velocity, independent of the strain related mechanisms cited above.

A discussion of the relative magnitudes and interrelationships of these effects is usually based on the continuum theory of finite deformations. When the assumptions of infinitesimal deformations of the material are abandoned, it is necessary to differentiate between the unstrained coordinate of a material point, a, and the deformed coordinate of that same point, x. The displacement is then defined as their difference,

$$\vec{u} = \vec{x}-\vec{a}. \qquad (1)$$

In this notation, the equations of motion assume the familiar form[5,6]

$$\rho\ddot{u}_i = \frac{\partial\sigma_{ji}}{\partial x_j}; \qquad (2)$$

However, the definitions of density, ρ, and stress, σ, must be generalized to reflect the deformation of the material and are defined by the series of equations:

$$\rho = \frac{\rho_o}{\mathrm{Det}(\partial x_p/\partial a_q)}, \text{ density of deformed material} \qquad (3)$$

$$\partial x_p/\partial a_q = \delta_{pq}+\partial u_p/\partial a_q, \text{ deformation gradients} \qquad (4)$$

$$\sigma_{ji} = \frac{(\partial x_j/\partial a_p)(\partial x_i/\partial a_q)t_{pq}}{\mathrm{Det}(\partial x_p/\partial a_q)}, \text{ stresses of deformed material} \qquad (5)$$

$$t_{pq} = C_{pqk\ell}\eta_{k\ell}+\tfrac{1}{2}C_{pqk\ell mn}\eta_{k\ell}\eta_{mn}, \text{ thermodynamic tension} \qquad (6)$$

$$\eta_{k\ell} = \tfrac{1}{2}\left(\frac{\partial u_k}{\partial a_\ell} + \frac{\partial u_\ell}{\partial a_k} + \frac{\partial u_i}{\partial a_k}\frac{\partial u_i}{\partial a_\ell}\right), \text{ Lagrangian (finite) strains.} \qquad (7)$$

Because of the complexity of the equations of motion in this form, it is illustrative to consider a special case. In particular, consider an anisotropic continuum with an orthorhombic symmetry (a symmetry with three orthogonal mirror planes which is characteristic of a rolled plate). Suppose for simplicity that a triaxial load is applied with stress components T_1, T_2, and T_3 along the three axes of the material. The abbreviated notation $T_1=T_{11}$, $T_2=T_{22}$, $T_3=T_{33}$ etc. is used in the following sections. In some cases, the full tensor notation will be required but these should be obvious from the context. Suppose, furthermore, that the relative values of these stress components are chosen such that the sample experiences a uniaxial strain, S_1, along the 1-axis. To the order of this calculation, the stresses are then given by

$$T_1 = C_{11}S_1 \tag{8a}$$

$$T_2 = C_{12}S_1 \tag{8b}$$

$$T_3 = C_{13}S_1 \tag{8c}$$

The ultrasonic velocities can then be obtained by seeking solutions to Eqs. (2)-(7) when small deformations are superimposed on the initial state. The computation is greatly simplified for the cases of propagation along the axes 1, 2, or 3 by the fact that the ultrasonic modes are either polarized parallel to the direction of propagation (longitudinal wave) or parallel to one of the orthogonal symmetry axes (transverse waves). Thus one seeks solutions of the form

$$\vec{u} = \hat{i}S_1a_1+\hat{j}e^{\sqrt{-1}(\omega t-ka_i)} \tag{9}$$

where $\hat{j}$ is a unit vector parallel to the j-axis.

Two definitions of velocity are commonly used in discussion of the relationship of deformation and wave velocity. The natural velocity, W_{ij}, of a wave propagating in the i-direction and polarized in the j-direction is defined as the initial (unstrained) path length divided by the transit time whereas the actual velocity, V_{ij}, is equal to the deformed path length divided by the transit time. The natural velocity definition is convenient in measurements of third-order elastic constants[5] and is given by ω/k. However, acoustoelastic measurements made on a material in an unknown state of deformation will yield the actual velocity. Restricting attention to transverse waves propagating along the 1-axis and polarized along the 2-axis or propagating along the 2-axis and polarized along the 1-axis, as shown in Fig. 2, one finds that the natural velocities are given by

$$\rho_o W_{12}{}^2 = C_{66} + (C_{166}+C_{11})S_1 \tag{10a}$$

$$\rho_o W_{21}{}^2 = C_{66} + (C_{166}+2C_{66}+C_{12})S_1 \tag{10b}$$

whereas the actual velocities are

$$\rho V_{12}{}^2 = C_{66} + (C_{166}+C_{66}+C_{11})S_1 \tag{11a}$$

$$\rho V_{21}{}^2 = C_{66} + (C_{166}+C_{66}+C_{12})S_1 \tag{11b}$$

Here the C's are the second and third order elastic constants in the abbreviated notation. Subtraction of Eqs. (11a) from (11b) illustrates the relationship cited in the introduction,

$$\rho(V_{12}{}^2-V_{21}{}^2) = (C_{11}-C_{12})S_1 = T_1-T_2 \tag{12}$$

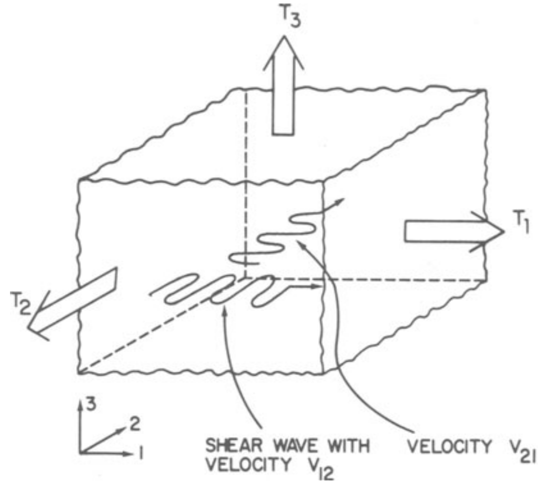

Fig. 2. Definition of shear waves whose velocities must be compared to separate stress from texture.

Note that a similar relationship is not satisfied by the natural velocities.

This simple relationship between the two transverse wave velocities and the difference in stress T_1-T_2 might of course be coincidental to the particular example of uniaxial strain. The discussion which follows illustrates that it is a more general result, as noted in the introduction. Consider the stresses, as given by Eq. (5), for the case of a wave propagating along the 1-axis and polarized parallel to the 2-axis. The stress associated with the wave has the form (full tensor notation)

$$\sigma_{12} = \frac{\left(\frac{\partial x_1}{\partial a_p}\right)\left(\frac{\partial x_2}{\partial a_q}\right) t_{pq}}{\mathrm{Det}(\partial x_p/\partial a_q)} \tag{13}$$

which directly contains the term

$$\left(\frac{\partial x_1}{\partial a_1}\right)\left(\frac{\partial x_2}{\partial a_1}\right) t_{11} \sim (1)\left(\frac{\partial u_2}{\partial a_1}\right) T_1 \tag{14}$$

in addition to those associated with second and third order elastic constants.

It could be argued that the static stress T_1 which appears directly in Eq. (14) could be replaced by the equivalent linear combination of static strains, and of course such a description would be correct. However, the interpretation of the stress term as a direct effect seems justified by the argument illustrated in Fig. 3. In the upper sketch, a sample under a static, uniaxial tension is shown. The dots represent material points which would fall on a cubic lattice if the load were removed. In the presence of the load, this unit cell deforms into a rectangular shape as shown by the dashed lines. There is no net force on this unit cell, as the effect of the static stress T_1 is equal and opposite on its two faces. However, as shown in the lower sketch, the presence of a transverse wave alters this situation. The unit cell is sheared and the applied stress, which remains parallel to the 1-axis in the material coordinates, a_i, no longer is parallel to the 1-axis in the deformed coordinates, x_i. For small amplitude waves, the angle of deviation between the direction of stress and the x_1 axis is equal to $\partial u_2/\partial a_1$. Since this slope changes with position, a net force in the z-direction acts on the unit cell and is proportional to $T_1\ \partial^2 u_2/\partial a_1^2$. As shown at the bottom of Fig. 3, the effect adds a term to the equations of motion with an effective shear wave stiffness equal to the stress. This effect is directly analogous to that which produces the previously noted[2,3] tension dependent velocity of a transverse wave on a stretched string. However, in this case it is superimposed on the usual elastic response of the solid.

A quite general discussion of these effects has been provided by Thurston[3]. He considers wave propagation in materials of arbitrary anisotropy which are under stress. Among the conclusions which are germane to the present discussion are the facts that:

1) The dynamic stresses depend upon material rotations (described by an antisymmetric tensor which does not enter stress-free, linear theory) as well as strains (symmetric tensor),
2) Effective elastic constants and wave speeds do not have all of the symmetry of the stress free case,
3) This symmetry reduction is an essential feature of a stressed material,
4) In particular, for a uniform stress and orthogonal pure mode directions in the presence of this stress (e.g., the case in which the principle axes of stress correspond with the symmetry axes of an orthorhombic plate).

$$\rho(V_{12}^2-V_{21}^2) = T_1-T_2 \tag{15}$$

5) For the most general case, one needs 27 independent coefficients to describe wave propagation, as compared to 21 for

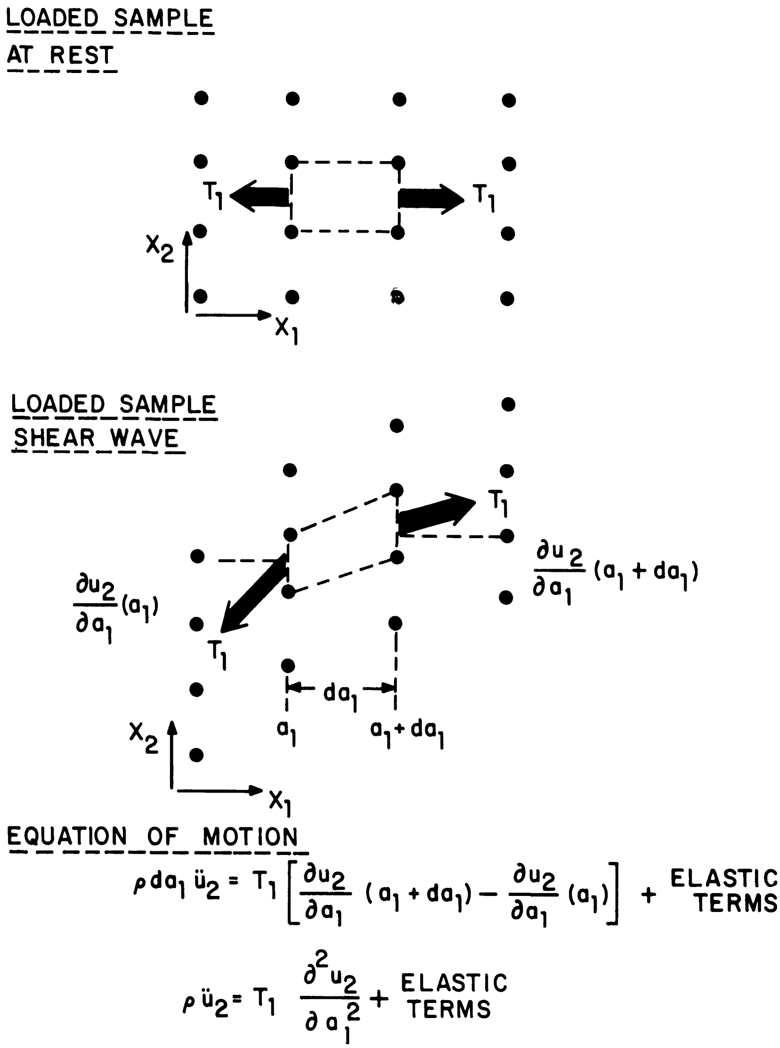

Fig. 3. Origin of direct influence of stress on ultrasonic velocity.

the unstressed case. The difference is the 6 components of stress.

It should be noted that for cases of lower than orthorhombic symmetry, or in which principal stress axes do not coincide with material symmetry axes, pure transverse modes may not exist for propagation along the 1 and 2 axes. In this case, V_{12} and V_{21} are not equal in the absence of stress and obviously Eq. (15) is no longer valid. More complex analysis is required for such cases.

APPLICABILITY OF THEORY TO ENGINEERING MATERIALS (POLYCRYSTALS) AND COMPARISON TO BIREFRINGENCE TECHNIQUE

Given the simple result of Eq. (15), which predicts that stress effects can be separated from elastic anisotropy effects in an elastic continuum, it is appropriate to consider the applicability of the theory to engineering materials such as metal polycrystals. The applicability of linear elasticity theory to wave propagation in such materials is seldom questioned. However, in the case of a second order effect, such as the one under discussion, some caution should be exercised. The following paragraphs represent a partial discussion of this question based on a review of the presently available literature.

A. Isotropic Case

Since the majority of studies of elastic wave propagation in metal polycrystals has been analyzed in terms of linear elasticity theory, this special case will be discussed first. Figure 4 contrasts the polarization and propagation directions of the two waves discussed above with those usually employed in the shear wave birefringence test. As noted in Eq. (15), for the case of a uniaxial stress, one expects a simple and direct relationship between the difference in the squares of the velocities and the stress. If one notes that $V_{12}{}^2-V_{21}{}^2 \simeq 2\sqrt{\mu/\rho}(V_{12}-V_{21})$, where μ is the linear shear modulus and it is assumed that the velocity differences are small, it follows that

$$2\mu\left[\frac{d(V_{12}-V_{21})/V_T{}^o}{dT_1}\right] \simeq 1 \qquad (16)$$

where $V_T{}^o=\sqrt{\mu/\rho}$ is the isotropic shear velocity in the absence of stress. This result is consistent with the commonly used theories for the propagation of elastic waves in isotropic solids[7] under stress. Equation (16) reveals that the constant of proportionality relating the fractional change in velocity difference, $\Delta(V_{12}-V_{21})/V_T{}^o$, to the applied stress T_1 is $(2\mu)^{-1}$. Since the shear modulus is usually well-known, and not strongly microstructurally dependent, this represents a simple relationship between the measurable velocity differences and the stress. This can be contrasted to the case for the shear wave birefringence test, for which the equivalent figure of merit would be

$$2\mu\left[\frac{d(V_{32}-V_{31})/V_T{}^o}{dT_1}\right] \not\equiv 1 \qquad (17)$$

The strong variability of this coefficient from one alloy to another has been pointed out elsewhere[1] and is illustrated in Fig. 5 with plots of values of the left hand side of Eq. (17) for a variety of steel and aluminum alloys. The pronounced contrast between these results and the constant value of unity for the left hand side of Eq. (16) is evident. The plot was prepared using the relationships

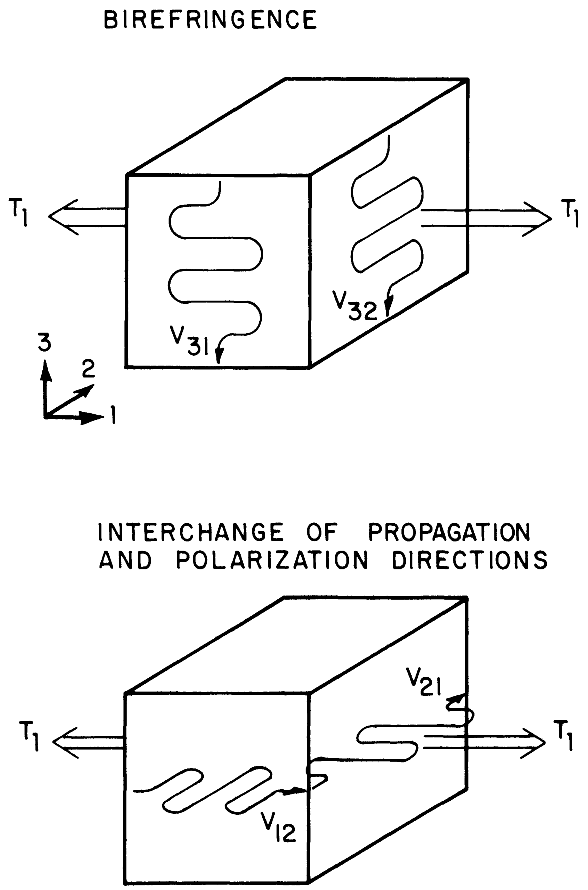

Fig. 4. Comparison of waves involved in ultrasonic birefringence test and in the interchange of propagation and polarization directions.

between velocity and stress given by Egle and Bray[8], with numerical values of the second and third order elastic constants from Hearmon's[9] compilation. This clearly establishes the reduction in sensitivity to microstructure of isotropic materials that can be

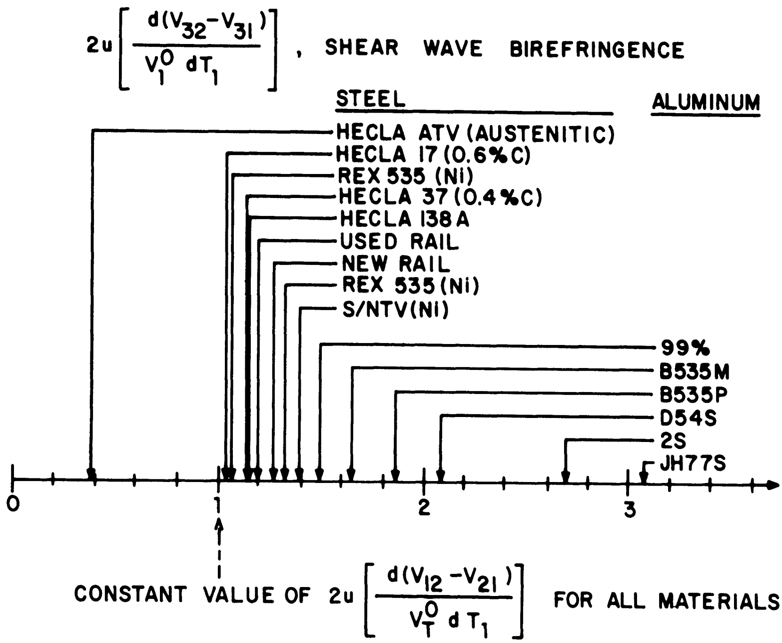

Fig. 5. Comparison of material variability of shear wave birefringence test and interchange of propagation and polarization direction test.

obtained by comparing the velocities V_{12} and V_{21} rather than V_{32} and V_{31}.

B. Anisotropic Case

For the case of anisotropic materials, there is little data available to directly test the predictions of Eq. (15). This situation probably exists because of the experimental difficulties inherent in measuring the velocities of waves propagating parallel to the direction of loading, coupled to the fact that such measurements are not required to determine third order elastic constants of many crystals[5]. An exemption is data obtained by Egle and Bray[8] on railroad rail, in which the changes in the velocities V_{12} and V_{21} with stress were measured independently. Figure 6 is a replot of the data reported in their paper showing the variation of V_{12}-V_{21} with stress observed on a sample machined from the head of a new railroad rail. Although they do not report the anisotropy of this sample, a few percent variation in the longitudinal velocity as a function of propagation directions would be consistent with the processing history of railroad rail and observations reported by the same authors on other samples[10]. Hence it must be asked whether $V_{12}-V_{21}=0$ when $T_1=0$. Absolute velocities were not reported by Egle and Bray. Hence this relationship was assumed in preparing Fig. 6

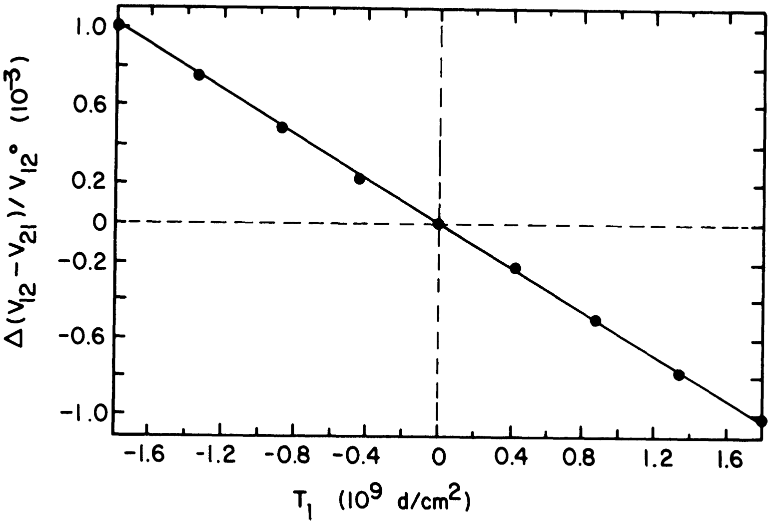

Fig. 6. Experimental measurement of V_{12}-V_{21} versus T_1 for railroad rail (after Egle and Bray[8]).

from the individual stress dependences of V_{12} and V_{21} reported by the authors. It would be justified if the rail head followed the predictions of continuum elasticity theory with either an orthorhombic symmetry (characteristic of a rolled plate) or an axially symmetric symmetry (characteristic of a drawn rod). As will be noted below, additional experimental work is required to establish the correctness of this assumption.

If this hypothesis is accepted, Table 1 presents the results of a direct test of Eq. (16) based on the data given by Egle and Bray[8]. The first two columns are values of the shear and Young's modulus measured by the authors (neglecting any anisotropy). The third column contains the observed coefficient of the stress dependence of

Table 1. Comparison of Measurements on Railroad Rails to the Predictions of Eq. (16)

Railroad Rail	μ (10^{12}d/cm^2)	E (10^{12}d/cm^2)	$d(V_{12}-V_{21})/V_T^{o}dT_{11}$ (10^{-12}d/cm^2)	$2\mu\left(\frac{d(V_{12}-V_{21})}{V_T^{o}dT_{11}}\right)$
New	0.799	2.071	0.652	1.042
Used (1907)	0.824	2.120	0.564	0.929

V_{12}-V_{21}, while the final column presents the results of using these experimental values to compute the L.H.S. of Eq. (16). The result is within 7% of the theoretically expected value of unity for both a new rail and a rail which had been in service since 1907. This agreement suggests that the predictions of the continuum elasticity theory, reflected by Eqs. (16) and (17), are applicable to at least some commercially important engineering materials which would be expected to have a significant degree of preferred orientation.

New experiments, presently in progress to test this conclusion in a more systematic fashion, are summarized in Fig. 7. An apparatus has been constructed which allows measurements of the speed of waves propagating both parallel and perpendicular to the direction of an applied uniaxial compression. Niobium has been selected as the material for the initial experiments because a)the single crystal is highly anistropic so that preferred orientation will produce large velocity anisotropies in polycrystals, and b) considerable experience is available for producing polycrystals with a variety of tailored grain shapes and orientations. The preliminary data tabulated at the bottom of the figure shows the longitudinal wave velocity shifts, observed for a value of T_1=-10kSI, are in reasonable agreement with theoretical expectations based on values of the third order elastic constants of niobium reported in the literature[11]. Shear wave measurements are presently in progress.

PRACTICAL MEASUREMENT CONFIGURATIONS

The above discussion identifies a specific physical measurement which can be used to directly measure stress in a polycrystalline metal with preferred orientations. However, since the measurement requires an interchange of the directions of polarization and propagation of a shear wave, practical configurations for application of these principles may not be obvious.

An example of one such configuration is shown in Fig. 8. Consider a rolled plate in a uniform, uniaxial stress oriented in a known direction, either parallel or perpendicular to the rolling direction. The above discussion indicates that this stress could be absolutely determined, independent of the degree of preferred orientation, by measuring the velocities of horizontally polarized shear waves (SH waves) propagating along and perpendicular to the stress axes. This can be accomplished in the plate geometry by using couplant free, EMAT transducers to excite the n=0, SH mode of the plate. Since the fields of this mode coincide identically with those of plane wave in an infinite medium, it should exhibit the same stress dependence of velocity. A similar approach would apply to the measurement of a uniform stress at the surface of a half-space. In this case the transducer would be used to excite the "surface skimming SH bulk wave". Separation of the two transducers by a rigid

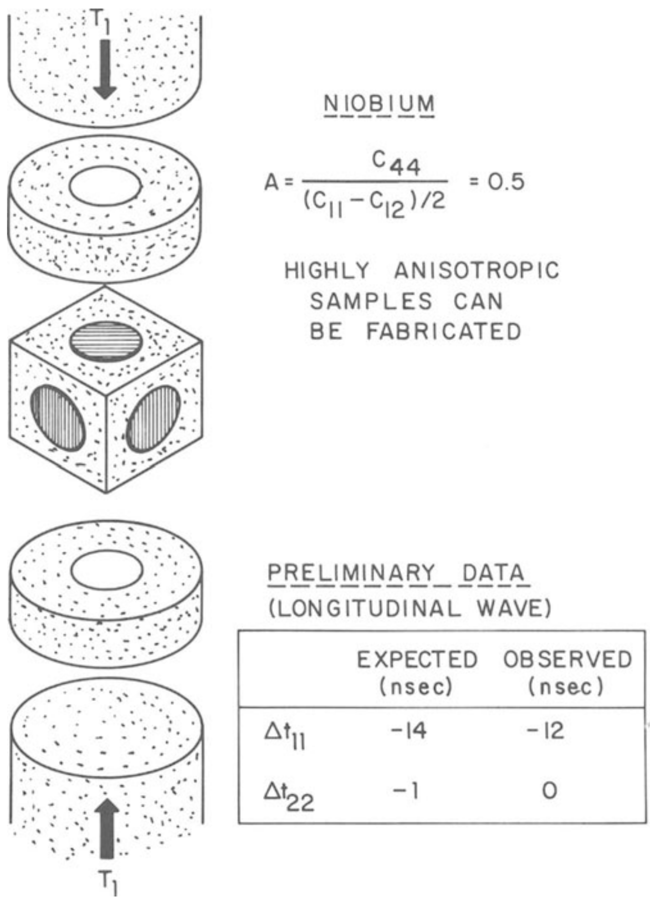

Fig. 7. Experiments to study effects of preferred orientation and microstructure on proposed stress measurement technique.

spacer of known length should allow the velocity along the two directions to be rapidly measured and compared.

SUMMARY

The fundamental difference between a stressed and an anisotropic material has been reviewed. It has been shown that, for a material

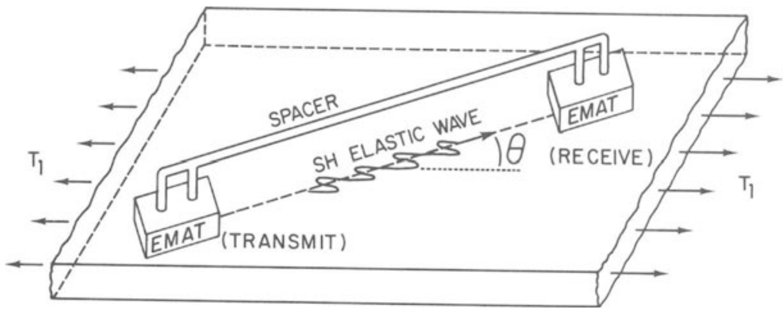

Fig. 8. Possible configuration for measuring a uniform stress in a textured plate.

of orthorhombic or higher symmetry (e.g., a rolled plate) with principle stress axes coinciding with the symmetry axes, the difference in principle stresses can be absolutely determined from measurements of density and wave velocity using Eq. (15). This result is of particular interest since no independent determination of texture is required and since no microstructurally dependent acoustoelastic constant must be known. The applicability of these predictions of continuum elasticity theory to polycrystalline metals is strengthened by reference to published stress dependences of ultrasonic velocities in railroad rail, and in-progress experiments to explore this question in greater depth are described. A practical measurement system to measure a uniform, uniaxial stress in a rolled plate having preferred orientation is proposed.

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